A qualitative approach for medial computation

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Abstract— **Many practical methods for the development of Medial axis transform have been aimed at addressing its instabilities and speeding up its computation. Here, the efficiency is improved using Voronoi based medial axis, with its dual Delaunay triangulation. A two dimensional image is taken as the input in this operation. The point set is used for computing its Medial Axis Trannsform. The output generated is a data structure of points of Medial Axis. It is used for comparing two shapes in computer graphics. It exhibits the desirable properties such as it allows trade-off between computation time and accuracy, making it well suited for applications in which an approximation of the medial axis is needed and computational efficiency is of particular concern. Also, the computational complexity depends on size of representation of the resulting medial, but not on the size of representation of the model. Finally, the approximated medial axis point densities in different areas are adaptive, based on the assumption that a coarser approximation in wide open area can still suffice the requirements of the applications**.

Index Terms—**Delaunay triangulation, Voronoi diagram, Medial axis.**

I. INTRODUCTION

The medial axis is an essential geometric structure in many applications and can be viewed as a compact representation for an arbitrary model. Medial axis of a shape provides a compact representation of it features and connectivity [1]. It has found applications in many fields like modeling, image processing, path planning, mesh generation, surface reconstruction, feature extraction, robot motioning and so on. In modeling, it can be manipulated to deform the boundary of the object. Also, this natural deformation can be used in computer animation. In image processing, it is used as a thinning method. In path planning, it serves as a guide because the path on it has a large distance from obstacles [2]. In mesh generation [3], [4], it provides a natural boundary to decompose the domain into sub domains which are easy to mesh. The same decomposition can be used in shape interrogation and simplification [5], injection molding simulation [6], surface reconstruction [7], feature recognition, etc.

A. Geometric details

The medial axis has been introduced by Blum [8] as a tool in image analysis. The *Medial Axis* of a two dimensional region is defined as the locus of the centre of all the maximal inscribed circle of skeletonization. Another related term for the same is a process for reducing foreground regions in a binary image to a skeletal remnant that largely preserves the extent. Also the connectivity of the original region while throwing away most of the original foreground pixels is maintained. The Medial axis transform of a rectangle has been shown in Fig 1.

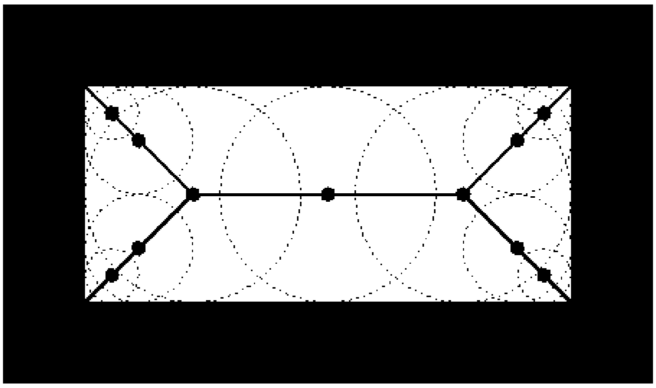

Fig 1: MAT of a Rectangle
(http://homepages.inf.ed.ac.uk)

In most commercial computer aided design systems, physical objects are represented at a low level as a collection of boundary topological elements; faces, edges, vertices, together with the geometry associated with each elements. Often this representation does not provide the shape information which is required for subsequent modeling applications. *Medial Axis* can be used as a complimentary representation which captures the geometric proximity of the boundary elements in a simple form. This led to an abstract shaped representation [9] known as *skeleton* which provides the unique description of shape which is of lower dimension than the original object. The medial axis together with the associated radius function of the maximally inscribed discs is called the medial axis transform (MAT).

A special kind of decomposition of a metric space, determined by distances to a specified family of objects in the space is the Voronoi diagram [1], [10–12]. These objects are usually called the Voronoi cell, namely set of all points in the given space whose distance to the given object is not greater than their distance to the other objects. It is a kind of decomposition. The segments of Voronoi diagram are all the points in the plane that are equidistant to the two nearest sites. The Voronoi vertices are the points equidistant to three or more sites.

The other one to improve the efficiency used here is Delaunay triangulation. It is obtained by connecting with the line segment for which a circle exists that passes through two points does not contain any other set in its interior or boundary. In general, a triangulation is a planar graph [8] with vertex set and straight line edges, which is maximal in the sense that no further straight line edge can be added without crossing other edges. The edges of the Delaunay triangulation are called the Delaunay edges [2]. As being the dual geometric [13] of the Voronoi diagram, comprises the proximity information inherent in a compact manner. It allows the well performance of the system in a more efficient manner that which can de adaptive to the changing environments.

The existing system does not have enough accuracy and the computational complexity is higher. In order to improve the system, enhancements are done in the proposed system. So, the computational data structure used in proposed system is Delaunay triangulation. The data structures are mentioned in the next section. Using these structures the computational complexity, speed and accuracy are intended to improve.

II. LITERATURE REVIEW

Tamal K Dey [1] in his work proposed medial axis of a shape provides compact representation of its medial axis and their connectivity. Here, exact computation is difficult, because medial axis is sensitive to small changes in shape. So, one way is to consider the point cloud representation of boundary surface of a solid and then attempt to compute an approximate medial axis from point sample. The other way is to approximate medial axis directly from Voronoi diagram. But, the difficulty is achieved by approximating medial axis straight from Voronoi diagram in scale and density independent manner with convergence guarantees. Also approximates medial axis from Voronoi diagram of a set of sample points.

Nina Amenta and Ravi Krishna Kolluri [14] explains the problem and then proposed the union of inner balls is a mean for efficient computation. The interior medial axis of union of balls is computed exactly. Also, the interior medial axis combines the characteristics of medial axis with combinatorial information. In many applications, it is useful to have a simplification of medial axis. The input given is sufficiently dense sample of points from the object surface, selection is made as a subset of Voronoi balls of the sample, the polar balls, as the union of balls representation. Also, the geometric error of the union, and of the corresponding power crust, and show that both representations are topologically correct as well. Thus, the result provide a new way for reconstruction from sample points. By construction, the power crust is always the boundary of a polyhedral solid, so polygonization is avoided. The union of balls representation and the power crust have corresponding piecewise-linear dual representations, which in some sense approximate the medial axis. A geometric relationship between these duals and the medial axis by proving that, as the sampling density goes to infinity, the set of poles, the centers of the polar balls, converges to the medial axis.

Balint Miklos, Mark Pauly and Joachim Giesen [10] mentioned and proposed medial axis of general union of points is not much efficient; on the other hand the medial axis of union of inner balls can be computed more efficiently and robustly. Here, dense sampling is used for the computation. The medial axis of the union of Voronoi balls [11], [15] centered at Voronoi vertices inside the shape and the Voronoi edges connecting them. In ordinary manner, two Delaunay triangulations are needed, whereas here only one Delaunay triangulation is needed. It works faster since it saves computation of restricted regular triangulation. The medial axis of union of balls immediately gives rise to approximate the medial axis of a smooth shape by the medial axis of the union of inner Voronoi balls and it involves the computation of the Voronoi diagram of the samples, the regular triangulation and the computation of the Voronoi diagram. Here, they show the medial axis of union of inner balls of a densely sampled smooth shape has a much simpler structure. This leads to a simpler and more robust way that only needs to compute one Voronoi diagram. More precisely, main result is that the medial axis of all inner Voronoi balls is simply the union of inner Voronoi vertices and inner Voronoi edges. Another structural result is the fact that for dense samples, the vertices of the union of inner balls are always sample points. Finally the result obtained is not only faster, also more robust.

Tamal K Dey, Hyuckje Woo, Wulue Zhao [16] discussed and proposed the quality achieved by method is surprisingly high as experimental results exhibit. The computation performed here becomes the major bottleneck in applications. It explains the approximation of medial axis of shape from a sample point. Here, the input is the coordinates of the sample points. As a result the approximation quality is limited by the input sample density. However, in geometric applications, the surfaces need to be derived from which samples need to be derived is known. The approach is achieved by first sampling [17] the surfaces appropriately and then modifying the medial axis approximation to exploit the known features of the input surfaces. The medial axis of a three dimensional shape embedded in three dimensions is the closure of all points that have more than one closest point on the shape boundary. The medial axis together with the distance to the boundary at each point is called the medial axis transform. The difficulty in computing medial axes can be attributed to their high algebraic degree and their sensitivity to small changes in shape. Voronoi diagrams have been shown to be useful for approximating the medial axes. In two dimensions the Voronoi vertices derived from a point sample of the boundary curve of a shape approximate the medial axis. Also, a Voronoi based method called medial was proposed that approximates the medial axis as a Voronoi sub complex.

Sungher Choi, Ravi Krishna Kolluri, and Nina Amenta [18] in their work proposed that there is a geometric relationship between the duals (union of balls and power crust) and medial axis. The transformation of medial axis representation of an object as an infinite union of balls. Approximation is done for medial axis transformation and its complement with finite union of balls. The input is dense sample of points [7] from object surface. Here, surface reconstruction is done from sample points. Union of balls and power crust are dual to each other. As sampling density goes to infinity, centres of balls converges to the medial axis. Hence, the approach is done by filtering Delaunay edges from Delaunay triangulation of the sample points since the method used has convergence guarantee. The approximation depends upon the sampling density and it approaches infinity. Also, experiments with different data sets [19], [20] support the concept. The medial axis as a compact representation of shapes has evolved as an essential geometric structure in a number of applications involving three dimensional geometric shapes. For most practical applications, a continuous approximation of the medial axis is required rather than a set of discrete set of points even if they lie close to the medial axis. The approach used here does not pay any attention to poles, but rather computes a sub complex from the Voronoi diagram that lies close to the medial axis and converges to it as sampling density approaches infinity.

G. Smogavec and B. ZAlik [21] presents the idea for constructing the approximation of a polygon's medial axis. It works in three steps. Firstly, constrained Delaunay triangulation of a polygon is done. After that, the obtained triangulation is heuristically corrected by inserting Steiner points. Finally, the obtained triangles are classified into three classes, thus enabling the construction of the polygon's medial axis. As shown by experiments, the obtained approximate polygon medial axis has better properties than those methods based on a Voronoi diagram.

In the present work a methodology is proposed and implemented for a simpler and faster medial axis construction of a two dimensional image. Point set information of the boundary is used effectively for Delaunay triangulation and then the triangles are further processed to extract the connectivity between adjacent ones to finally shape the axis. This differs from the previous works from the fact that an iterative sampling approach is employed to eliminate the unwanted points and hence reduce the complexity. The work is expected to be taken forward to further reduce the complexity. Also the strategy used here for two dimensional shapes can be further extended to three dimensional shapes by dividing it into small slices of two dimensional areas. The methodology is expected to bring down the time that is required in shape searching and other areas where the use of medial axis transform is involved.

III. METHODOLOGY

A. *Representation of the proposed methodology*

The methodology is shown in the fig 2. The two dimensional image is first taken as the input and its edge is detected using an edge detection algorithm. The boundary information is then stored as pixels of a particular identified intensity value. Then a scanning of the image is done to get the geometric coordinates of the edges. This point set is then further used as an input for the upcoming Delaunay triangulation. An iterative triangulation algorithm is used for dividing the point set space into triangles. The algorithm used to compute the triangulation of the point set is described below as series of steps.

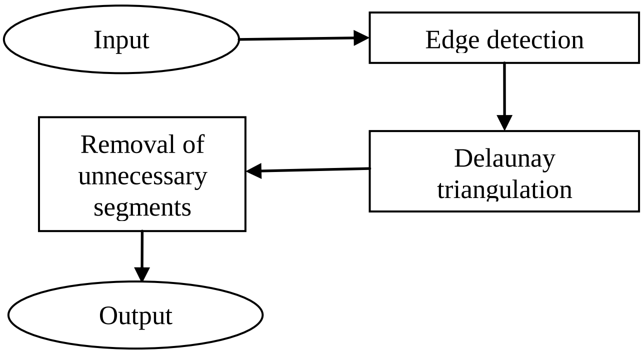

Fig 2: Framework for the generation of medial

B. *Steps for the generation of Delaunay*

1. Input: Point Set
2. Output: triangle list
3. Determine the super triangle (Triangle that contains all the points)
4. Add the super triangle to the triangle list item.
5. For each sample point in the Vertex list item initialize the item for each triangle currently in the triangle list.
6. If the point lies in the triangle circum circle then
7. Remove the triangle from the triangle list
8. End if
9. End For
10. Delete all doubly specified edges from the edge buffer
11. This leaves the edges of the enclosing polygon only
12. Add to the triangle list all triangles formed between the point and the edges of the enclosing polygon
13. Remove any triangles from the triangle list that use the super triangle vertices
14. Remove the super triangle vertices from the vertex list.

The speed of computation of this algorithm is O (n^2).

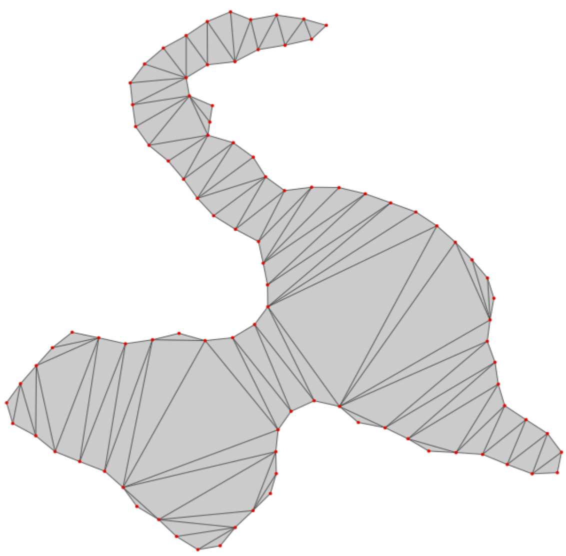

Fig 3: Delaunay triangulated boundary

Fig 3 shows a map represented by its boundary. The point set of the boundary is subjected to Delaunay triangulation as shown. The nature of the triangles in the Delaunay Triangulation are computed and characterised based on its properties such as area, circum centre and centroid to deduce the medial axis transform [5], [22] from the same. Initially the medial axis is generated in a crude way. Then it is further pruned and refined to get the final medial. Edges are checked and that are outside the polygon are removed and can be classified as external medial axis segments. The internal segments are further refined to get the final medial axis output. This then stored as a data structure with the continuity dictated by the successive points. Along with each point the radius information of the circle that rolls through it is stored. The medial axis transform [8], [24],[24] is further stored as an edge data structure. It is then validated by reconstructing the boundary and the boundary is then compared with the original image boundary using the computation of distance.

IV. RESULTS AND DISCUSSION

The methodology described above was tested on images and one example is presented below. Points, edges and triangle were stored as structures. The image was taken as a jpeg file and then it was subjected to edge detection operation using canny edge [25] detection operator .The edge was then set as a boundary with pixels of a particular intensity. A scan operation was then performed to identify the geometrical coordinates of the boundary pixels. The geometrical coordinates were further normalized with respect to a fixed origin in the image.

The points are then subjected to Delaunay triangulation as shown in Fig 5 .The white lines show the Delaunay edges which are the dual for the Voronoi representation [26]. This triangulation contains information about the medial axis. The next task would be to extract the axis using the different properties of triangle like circum centre and area. The red lines shown in Fig 5 are the initially constructed axis. This is then pruned by doing checks on the image to find whether the segments are inside or outside the boundary of the image. The pruned image is then shown in Fig 6. The final medial axis is shown in Fig 7 inside the closed boundary of the image.

Fig 4: A Silhouette image of a hand

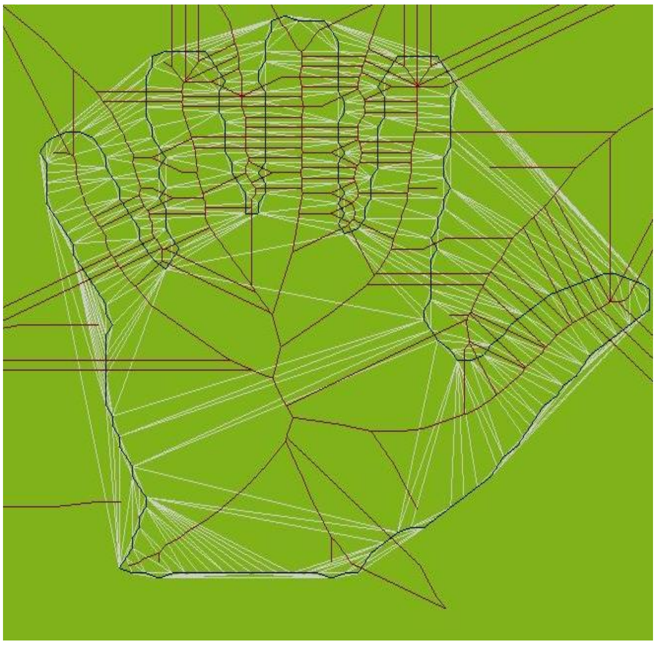

Fig 5: Triangulation is done

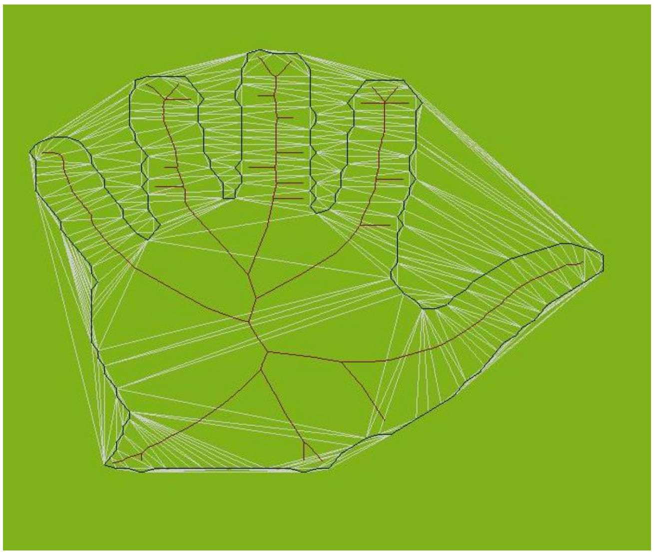

Fig 6: Unwanted segments removed

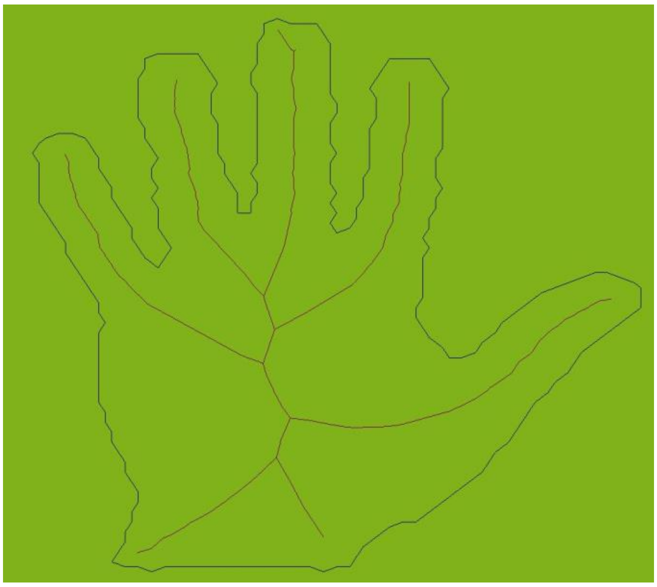

Fig 7: The final medial axis

Another image is subjected to the same operation as shown in Fig 8, Fig 9, and Fig 10. The branch points can be seen in Fig 10. They were identified in the axis using the property of the triangle. The medial axis transform was subjected to reconstruction which yielded reliable results.

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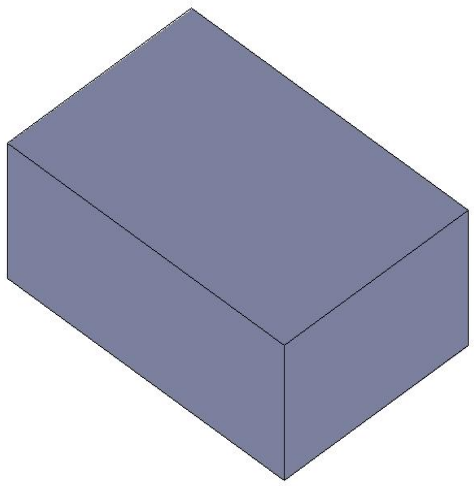

Fig 8: A Silhouette image of a cuboid

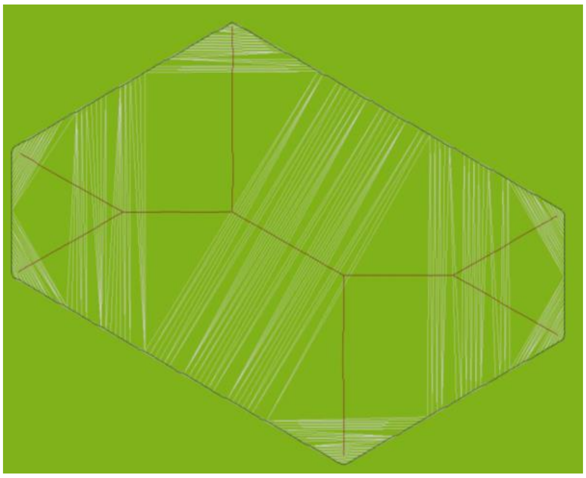

Fig 9 : Subjected to triangulation

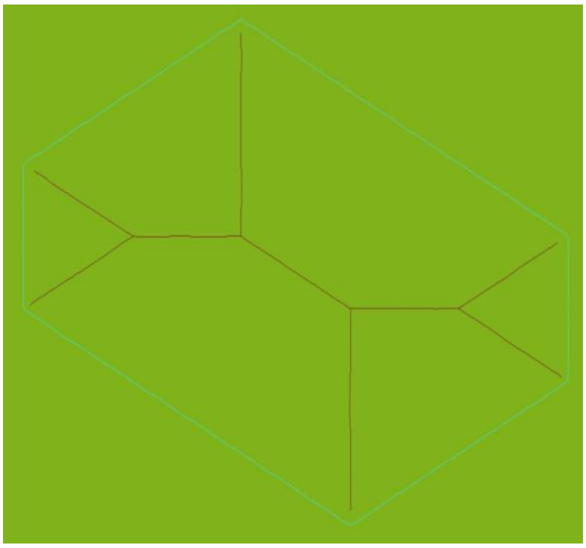

Fig 10: The final medial axis

V. CONCLUSION

A novel method was developed and tested to compute medial axis .It was found to reduce complexity and improve accuracy to a good extent by selectively choosing the points needed for operation. The work would be extended to further improve the results. The same strategy is planned to be extended to three dimensional as well.

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