**Volume Constrained Polyhedronizations of Point Sets in 3-Space**

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**Minimum Volume Polyhedronization of Platonic Point sets**

**Polyhedronization**

- **Definition**
  “Given a finite set of points in $R^3$, polyhedronization deals with constructing a simple polyhedron such that the vertices of the polyhedron are precisely the given points.”

- **Applications**
  - Molecular polyhedron structure synthesis.
  - Boundary representation of input points in Computer Graphics, Computer Vision & Distance Image Processing.

**Algorithm**

- **RAA_MINVP-Randomized Approximation Algorithm**
  Let $S=\{p_0, p_1, \ldots, p_{n-1}\}$ denotes the point set.

  **Initialization**
  Select four points uniformly at random from $S$ and form an initial tetrahedron $P$.

  **Iterations**
  In each iteration, it chooses one point $q$ uniformly at random from $S \setminus P$. Determines the position of $q$ relative to the previous polyhedron $P$ and does one of the following.

  1. $q$ lies interior to $P$? -> exclude from $P$, the largest volume tetrahedron that makes with any of the visible faces of $P$.
  2. $q$ lies exterior to $P$? -> add to $P$, the smallest volume tetrahedron that forms with any of the visible faces of $P$.
  3. $q$ lies on an edge of $P$? -> divide the adjacent faces of that edge into four new faces by including $q$ as the common vertex of all the four faces.
  4. $q$ lies on a face of $P$? -> divide the face into three new faces by including $q$ as the common vertex of all the three faces.

  **Termination**
  Once the iterations are completed, algorithm returns the final polyhedron (The set of faces).

- **RAA_MAXVP**
  The initial polyhedron is the convex hull of $S$. The iterations are pretty much similar to the iterations of RAA_MINVP. Both differs only in steps 1 & 2.

  - $q$ lies interior to $P$? -> exclude from $P$, the smallest volume tetrahedron that makes with any of the visible faces of $P$ and vice versa.

**Problem Statement**

- **FACE problem by S.P Fekete [FP93]**
  “Let $2 \leq d$ and $1 \leq k \leq d$. Given a finite set $S$ of points in $d$-dimensional Euclidean space. Among all simple polyhedra that are feasible for vertex set $S$, find one with the smallest volume of its $k$-dimensional faces.”

- **Minimal(Maximal) Volume Polyhedronization (MINVP (MAXVP))**
  “Given a finite set $S$ of $n$ points in $R^3$, find the simple polyhedron with the smallest (largest) volume from all the simple polyhedra (having triangular faces) that are feasible for the vertex set $S$.”

**Results**

- Approximate MINVPs generated for Prismatic Point Sets.
  - MINVP
  - MAXVP
  - Point set 50
  - Point set 75
  - Point set 100
  - Approximate MINVPs & MAXVPs generated for Point Sets of different sizes.

- Optimal MINVPs and/or MAXVPs generated for Prismatic Point Sets.

**Future Work**

- To address the following questions:
  - What are the performance guarantees of both the algorithms?
  - Does there exist an input configuration for which the approach fails for every possible ordering of points?

**References**


Suggestions/Comments? - please mail to - emry01@gmail.com, jijupnair2000@yahoo.co.in