Concave and α -Hull of a set of freeform planar curves

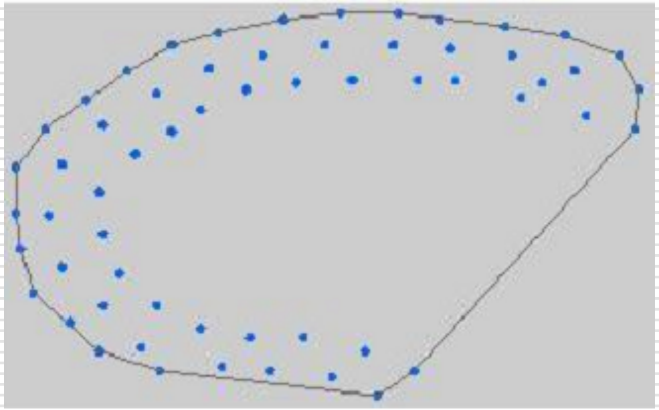
A.V.VISHWANATH

R.ARUN SRIVATSAN

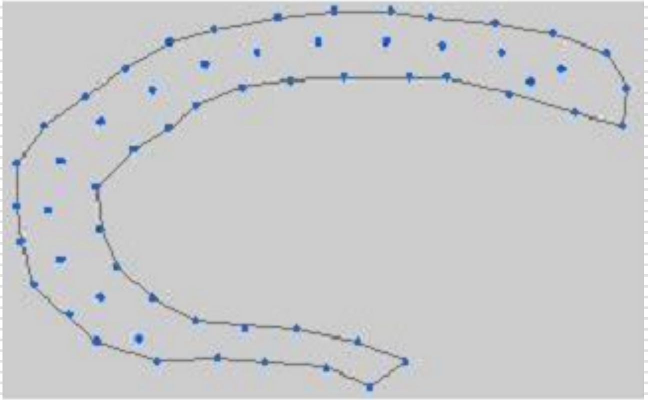
M.RAMANATHAN

Department of Engineering Design
Indian Institute of Technology Madras

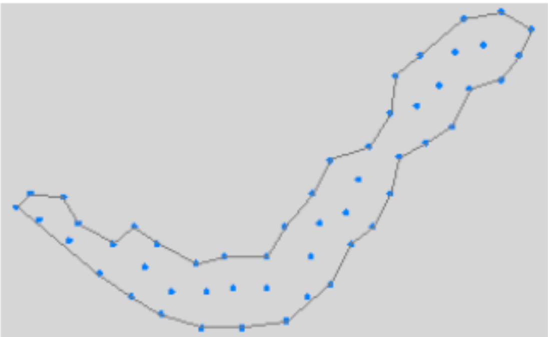
Region occupied by a set of points



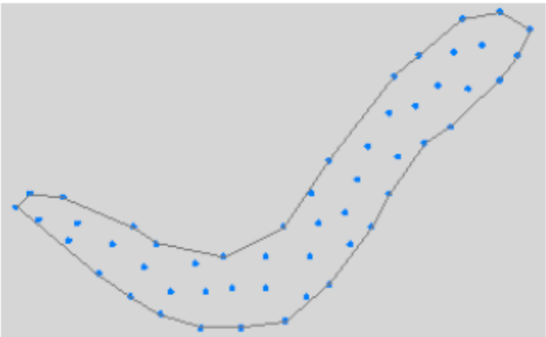
Convex hull



Region occupied by a set of points



Coarse concave hull



Smoother hull

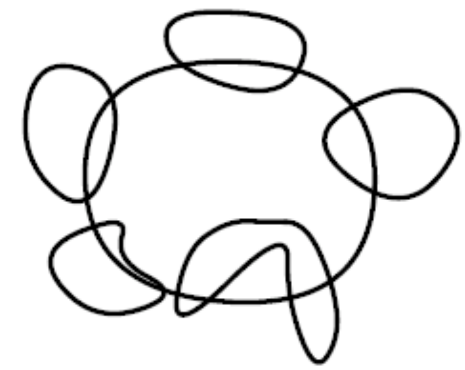
Contributions

- Definition of concave hull
- Algorithm to compute concave hull for a set of curves
- α-hull of set of freeform curves
- Comparison with concave hull

Concave Hull of a set of planar curves

Definition:

Concave Hull of a set of curves is the enclosing concave curve with the smallest area.



Nature of curves:

- C^1 continuous
- Non self intersecting
- Closed
- Simply connected
- Interior lies to left when traveled anti-clock wise along the curve
- No straight line portion

Concave Hull of a set of planar curves

Cases considered when two curves are involved

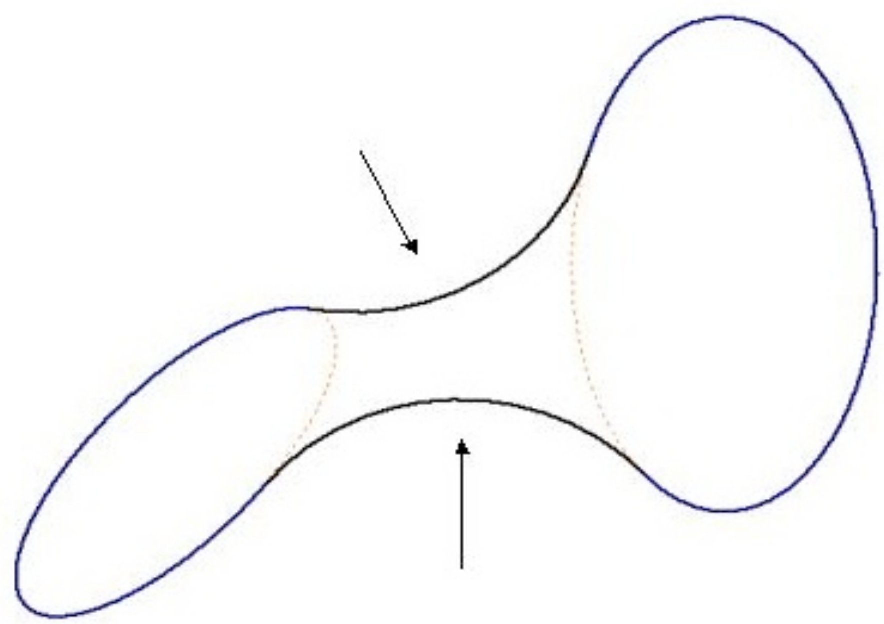
- One curve lying completely inside another
- One curve lying completely outside another
- Intersecting curves

Concave Hull of a set of planar curves

One curve is completely inside another

Concave Hull of a set of planar curves

One curve is completely outside another



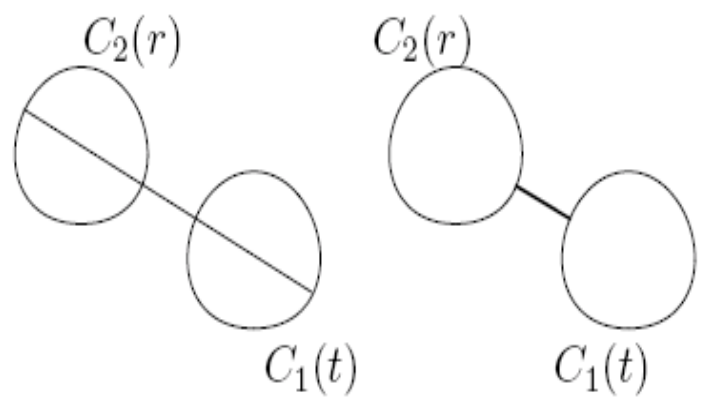
Concave Hull of a set of planar curves

One curve is completely outside another

Lemma 1:

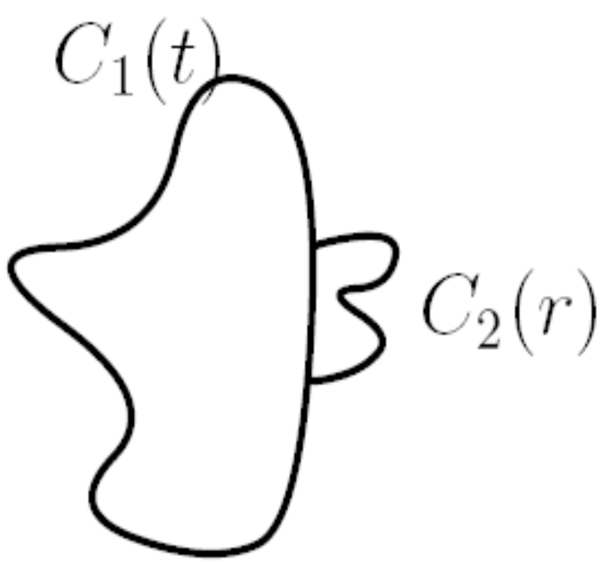
The distance between two closed C^1 non intersecting curves is minimum only when the normals of the corresponding points are opposite to each other

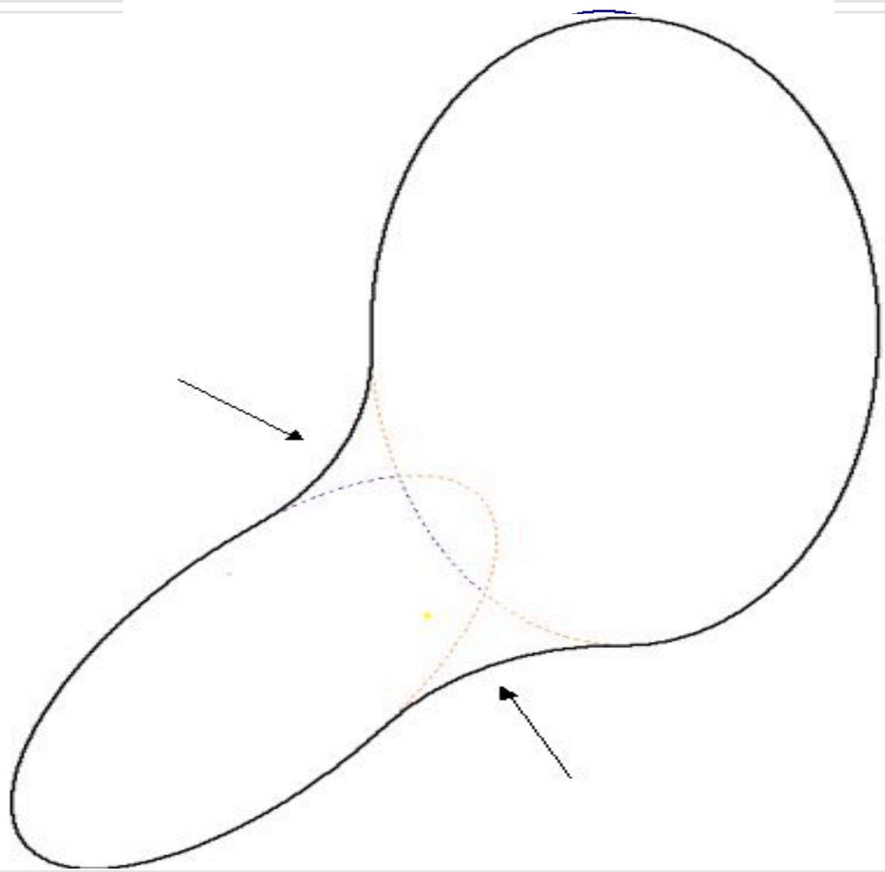
$$\left\langle C_1'(t), C_1(t) - \frac{C_1(t)+C_2(r)}{2} \right\rangle = 0,$$
$$\left\langle C_2'(r), C_2(r) - \frac{C_1(t)+C_2(r)}{2} \right\rangle = 0.$$



Concave Hull of a set of planar curves

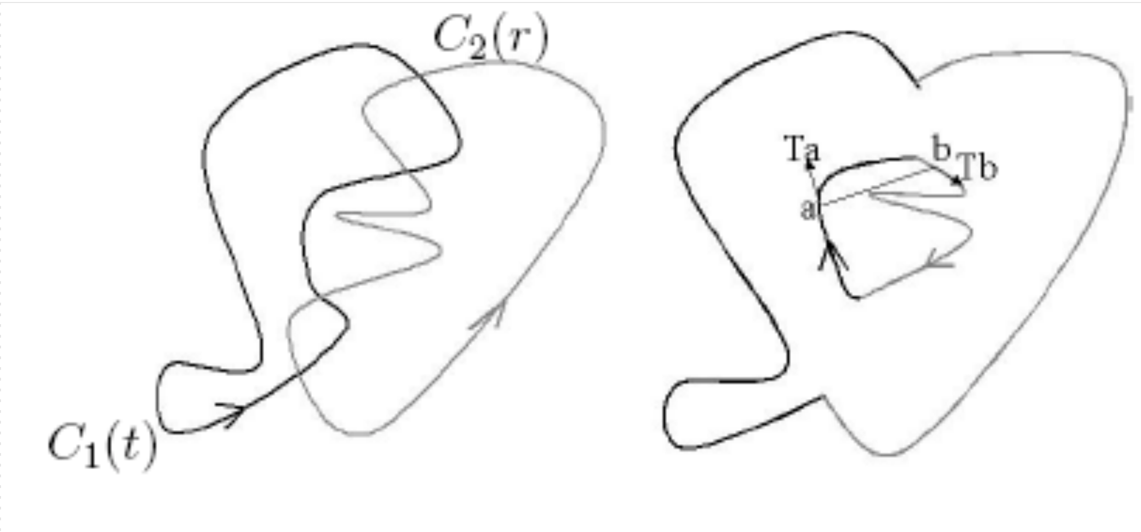
- Two intersecting curves





Concave Hull of a set of planar curves

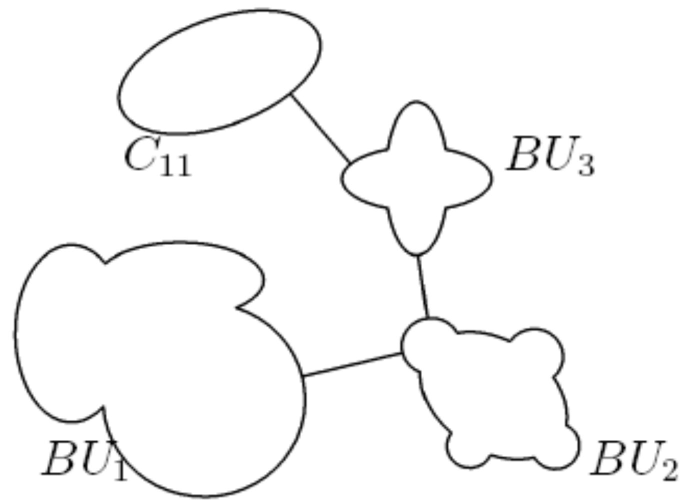
- Inner loop formed due to intersecting curves



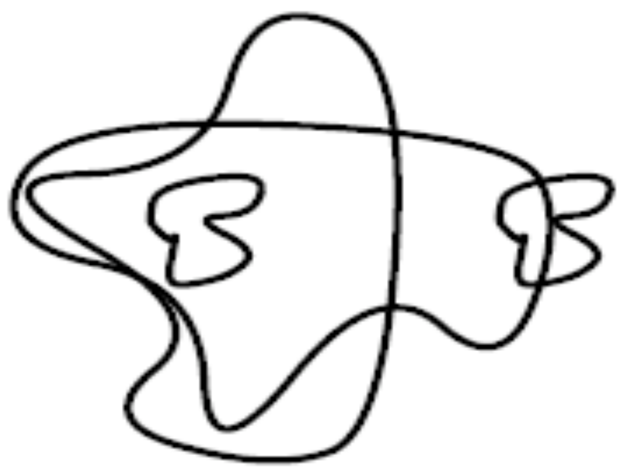
Concave Hull of a set of planar curves

Algorithm $CCVHULL(C = C_1, ..., C_n)$

Determine curves that intersect each other.
Eliminate curves that lie completely inside.
Find sets of curves that intersect. Let S_i denote each set, and each S_i will consist of curves from C.
for each set S_i of intersecting curves **do**
  Perform Boolean union.
  Represent each Boolean union as a single curve (say BU_i).
  Eliminate interior loops.
end for
Let $C' = \{\{BU_i\} \bigcup \{C - \{S_i\}\}\}$.
for each curve in in C' **do**
  Compute MAP to all other curves in the set.
end for
if All curves in C' are outside each other **then**
  Use curves as nodes and MAPs' as distances, find the minimum spanning tree (MST).
  Return MST as concave hull.
else
  Return C' as concave hull.
end if



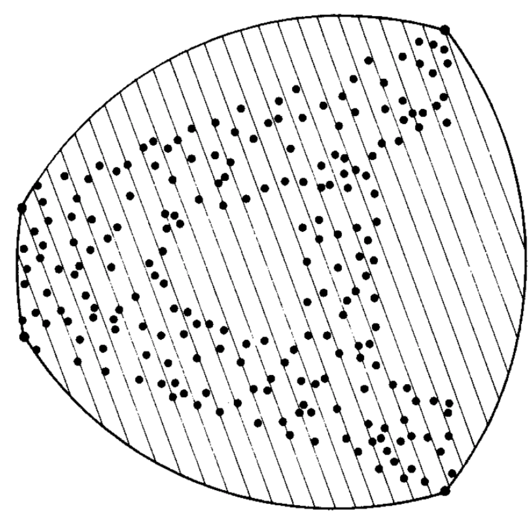
Some results for concave hull



α-Hull of a set of planar curves

- Positive α-Hull ($\alpha > 0$)

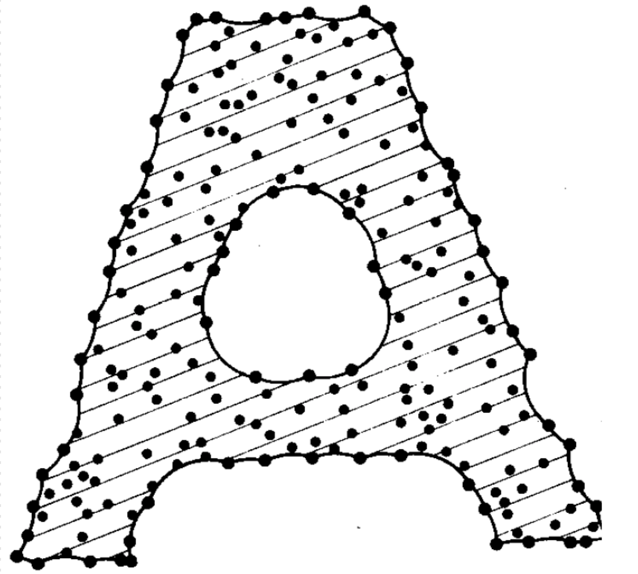
The α-hull of S is the intersection of all closed discs with radius l/α that contain all the points of S.



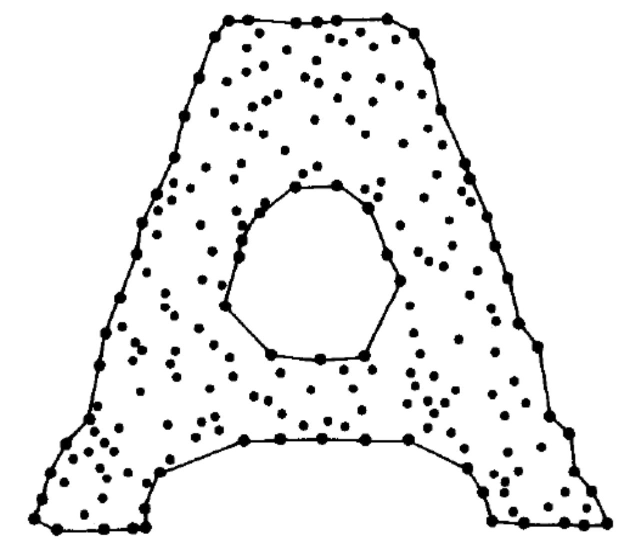
α-Hull of a set of planar curves

- Negative α-Hull ($\alpha<0$)

The α-hull of S is the intersection of all closed complements of discs with radius $-1/\alpha$ that contain all the points of S.



α-Hull



α - Shape

α-Hull of a set of planar curves

- Extension of α-Hull of points to curves

o α-shape (discrete counterpart of α-Hull) obtained from delaunay triangulation.

o Delaunay of curves not well defined.

o Hence α-hull of curves is explored in terms of Voronoi diagram (Dual of delaunay triangulation) of set

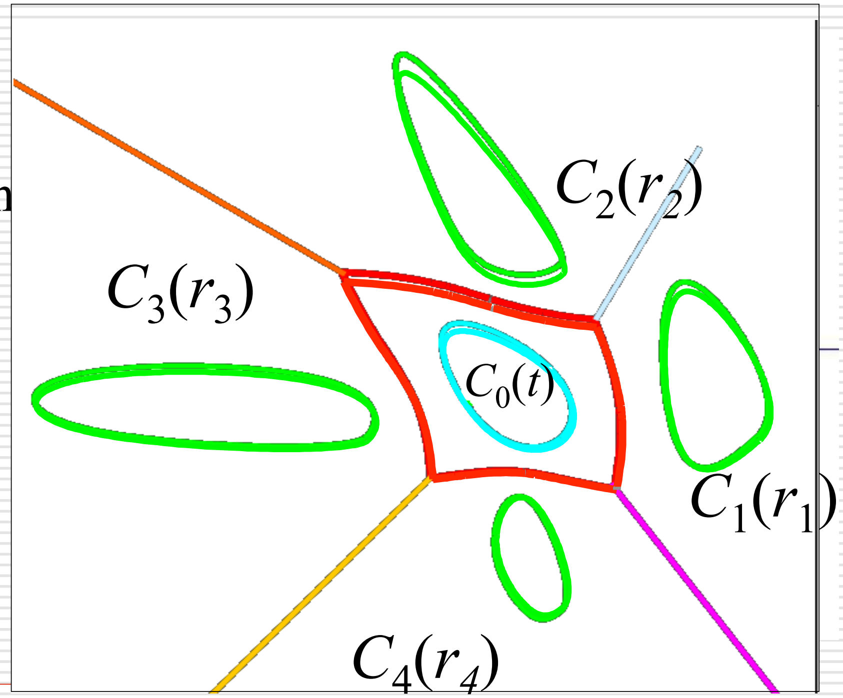
α-Hull of a set of planar curves

- Voronoi Cell (Closest point)

  The Voronoi cell of a point P_i in S is the set of all points closer to P_i than to P_j , $\forall P_j \in S$ and $i \neq j$.

- Voronoi Diagram

  The Voronoi diagram is then the union of the Voronoi cells of all the points in the set.

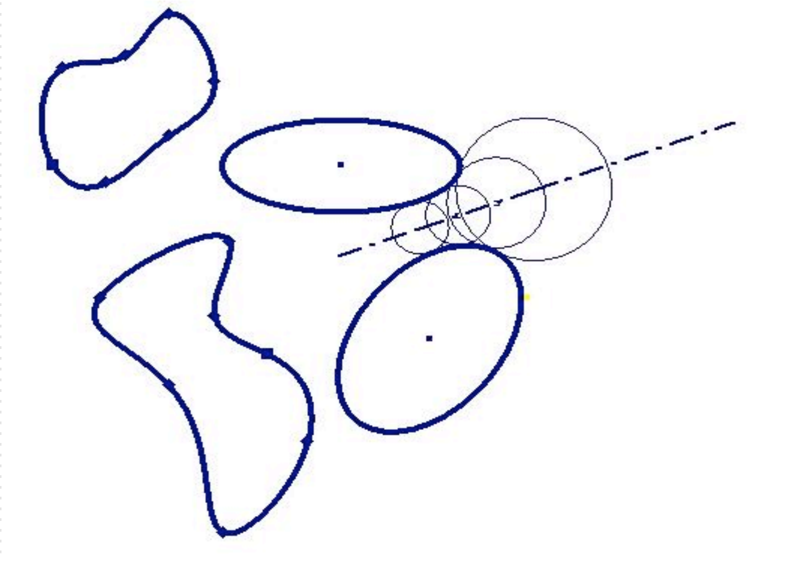


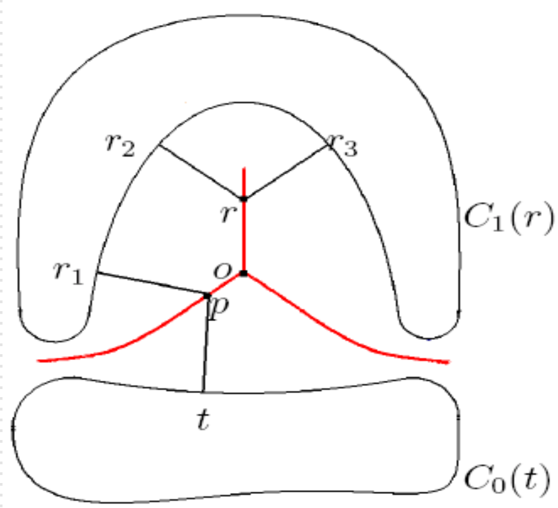
α-Hull of a set of planar curves

- Extension of α-Hull of points to curves

Lemma:

The centres of the α-disc have to lie on the exterior Voronoi diagram of the set of curves for all $\alpha < 0$.





Relation between α-Hull and concave hull of curves

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Lemma

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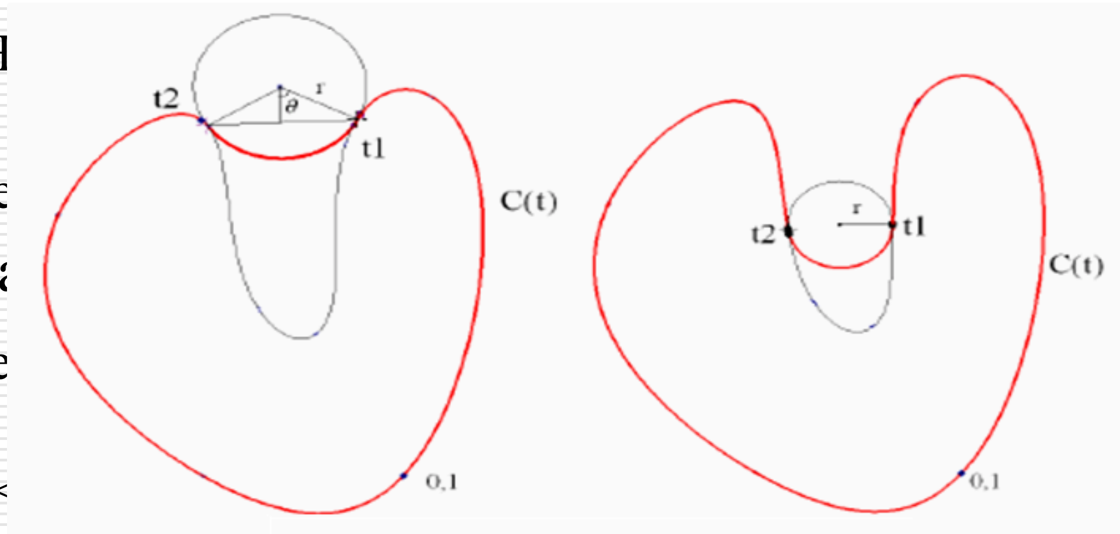
$\forall \alpha$ <



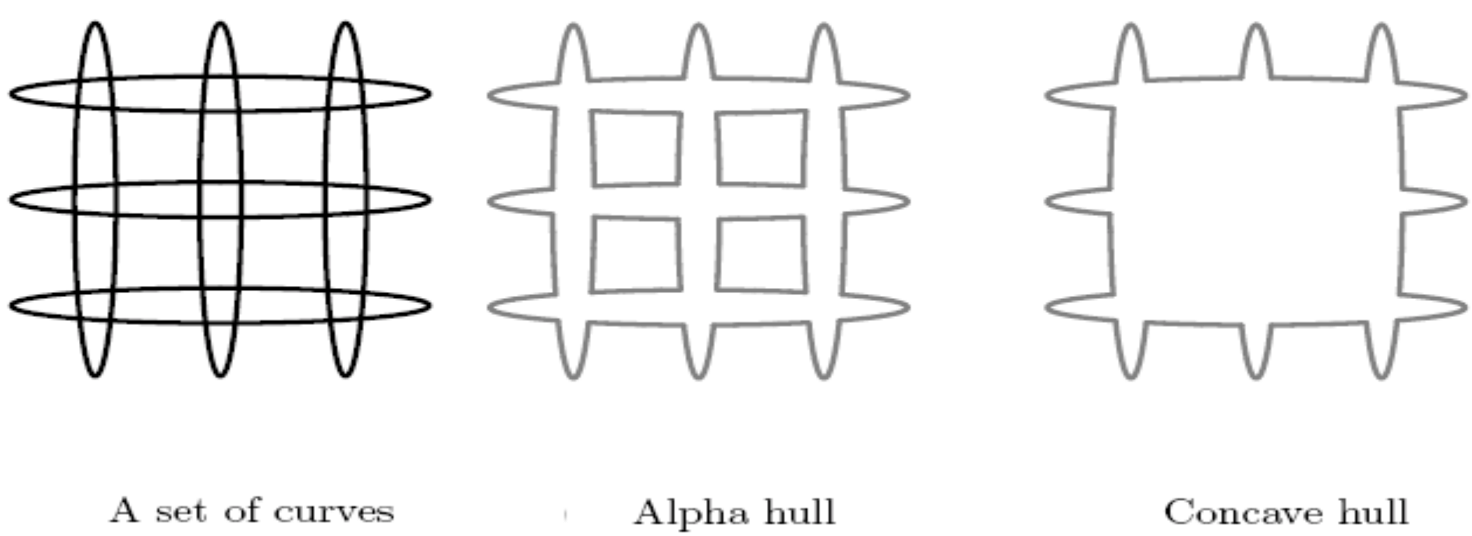
$$Length_{\alpha} = t1 + (1 - t2) + 2r\theta$$

- α - hull with $\alpha = -\infty$ is used.

Relation between α-Hull and concave hull of curves

Lemma:

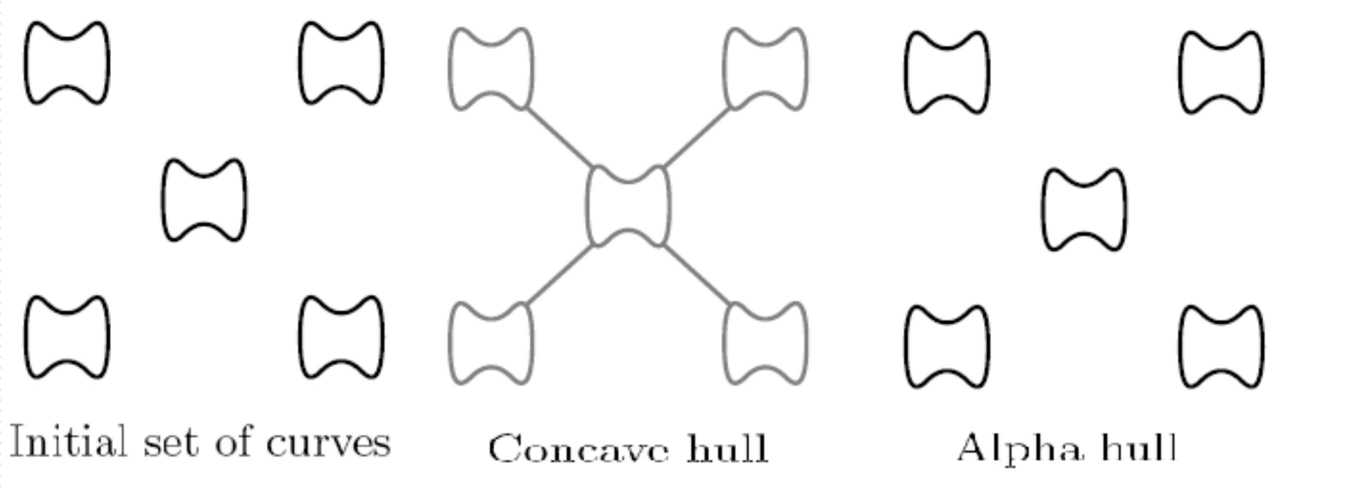
For intersecting curves, boundary of α-Hull is superset of concave hull



Relation between α-Hull and concave hull of curves

Lemma:

Boundary of the α-hull is a subset of the concave hull when the curves lie outside each other.



Relation between α-Hull and concave hull of curves

When a set consists of both the categories, it is difficult to ascertain a relationship between both

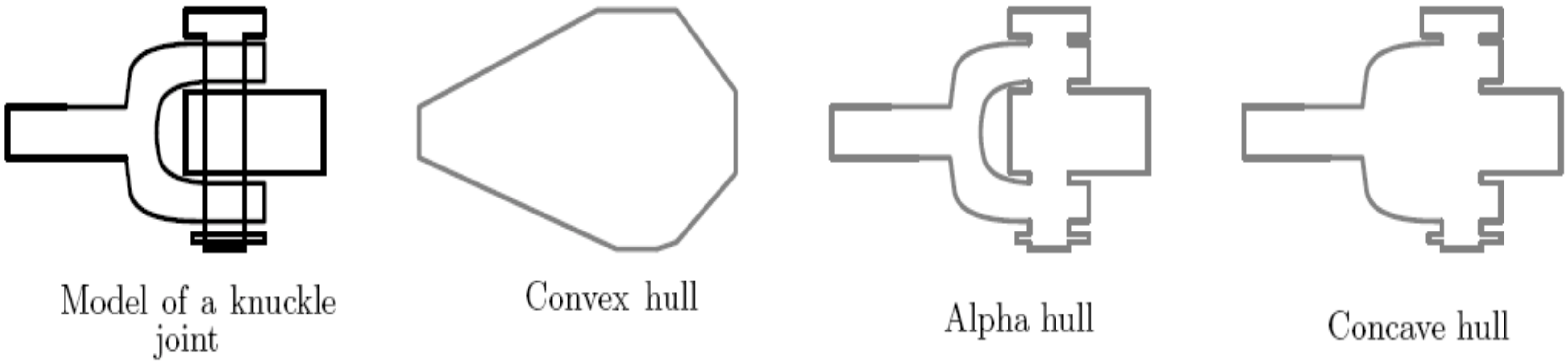
Input set of curves.

α-hull of the set of curves with $\alpha = -\infty$

Concave hull of the set of curves.

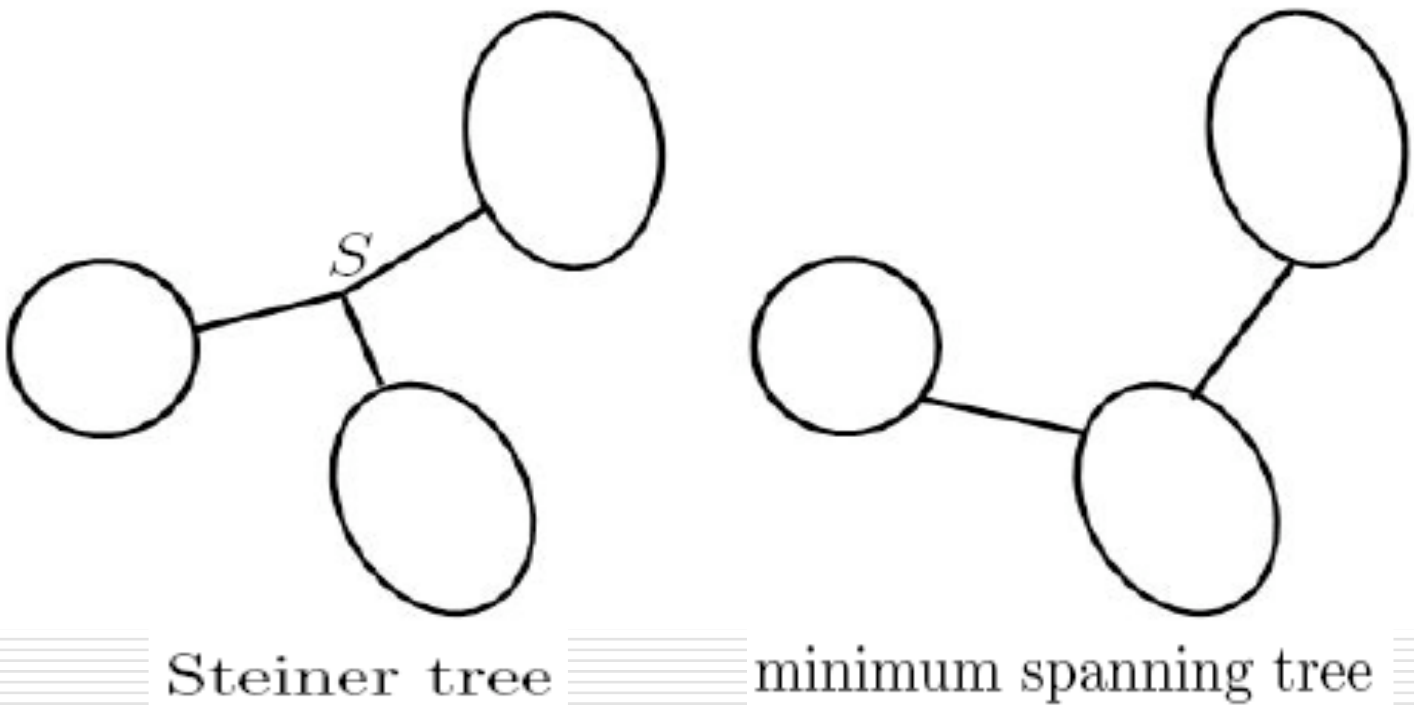
Relation between α-Hull and concave hull of curves

- Some Examples



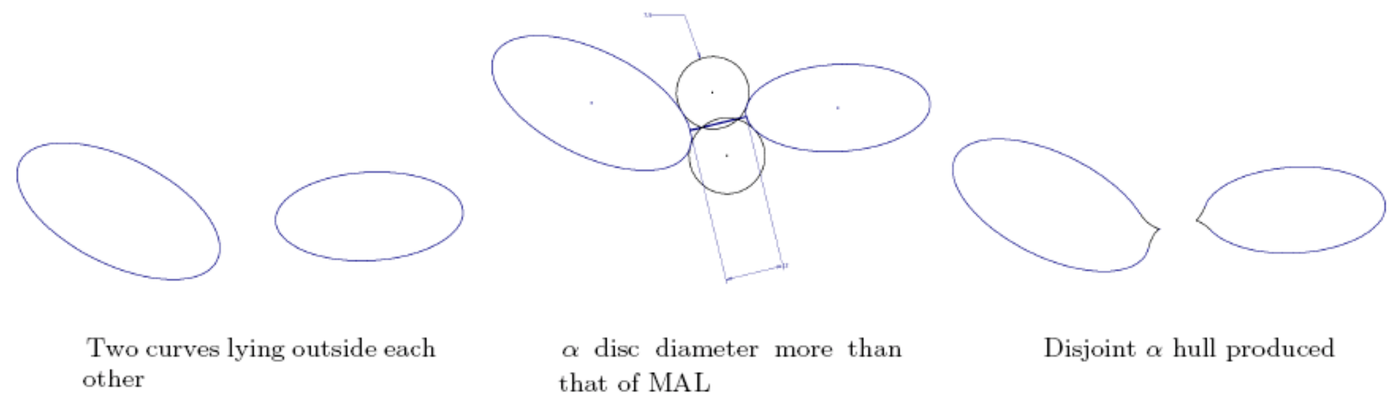
Discussions

- Steiner tree approach



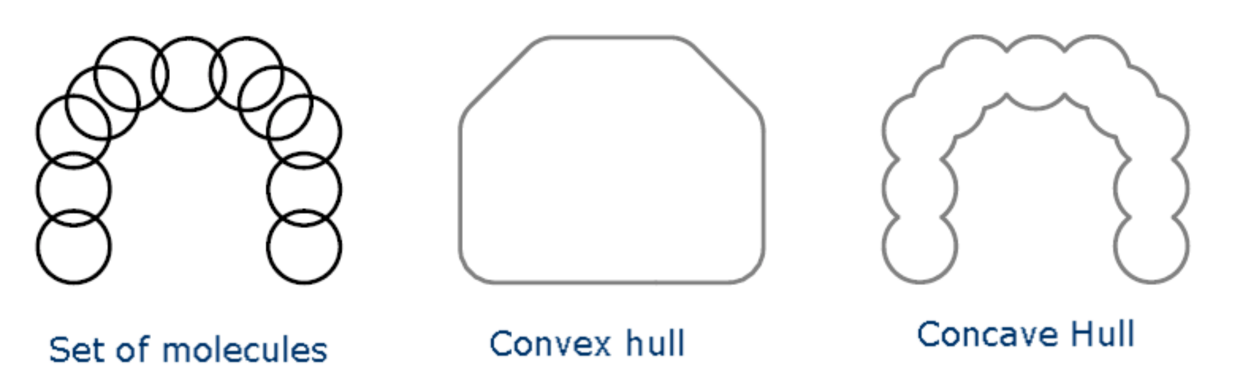
Future work

- Extension to 3-Dimension
- Generating a non disjoint α – hull for a set.



Potential Applications

- Concave hulls can be used for molecular shape matching
- Geographic information processing.
- Fenc



Conclusion

- Developed a definition for concave hulls of a set of curves
- Showed that it is the Boolean union for intersecting curves and MAL is computed for non –intersecting curves
- Relation between concave and α- hull is established