Tutorial #1, due in class on August 12, 2011

Ground Rules

- Consultation with your team-mates, TAs or the Instructor is encouraged. However, each student is expected to write out and hand in his/her own solutions.
- Please turn in solutions to all questions today.
- **Grading**: 1 point for fully correct solution. No partial grading.

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1. Consider a stress state at a point given by the following components (in MPa) in a basis $\mathbf{e}_i (i = 1, 2, 3)$:

$$\begin{bmatrix} 300 & -200 & 450 \\ -200 & 700 & -200 \\ 450 & -200 & -400 \end{bmatrix}$$

   - Compute the hydrostatic stress and the deviatoric stress tensor, $\mathbf{S}$
   - Compute the second invariant (J_2) of $\mathbf{S}$. Note that J_2 will be extensively used in formulating the von Mises yield criterion for fully-dense metals.

2. Consider the stress at a point in the interior of a solid loaded by external tractions. Suppose that the eigenvalues of the stress tensor at this point are σ_1, σ_2, and σ_3 such that $\sigma_1 \geq \sigma_2 \geq \sigma_3$. Then, the stress components in the cartesian basis formed by the unit eigenvectors $\hat{\mathbf{n}}^{(i)}, i = 1, 2, 3$ are

$$\sigma = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

   (a) Consider a second set of basis vectors that are obtained by rotating the $\hat{\mathbf{n}}^{(1)}$ and $\hat{\mathbf{n}}^{(3)}$ vectors through an angle θ about the $\hat{\mathbf{n}}^{(2)}$. Write down the components of the transformation matrix that relates the new coordinates to the original eigenvector basis. Assume θ to be positive counter-clockwise, as is the usual convention.

   (b) Compute the stress tensor components in the new coordinate system.

   (c) What is the angle of rotation that maximizes the magnitude of the shear stress and what is this maximum magnitude?

3. Given $\sigma_{11} = 3$ MPa, $\sigma_{12} = 2$ MPa, and $\sigma_{22} = 1$ MPa at a point, find

   (a) the maximum and minimum normal stresses at the point and

   (b) the maximum magnitude of shear stress at this point and the planes on which this shear stress acts.

   (c) Show using Mohr's circle, the directions of maximum and minimum normal stresses.