Tutorial 10

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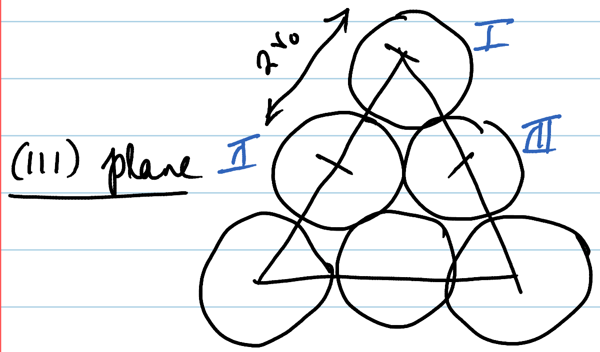
1) On a close-packed plane in an FCC crystal, we have the following arrangement of atoms.



$r_0 \rightarrow$ equilibrium spacing between atoms

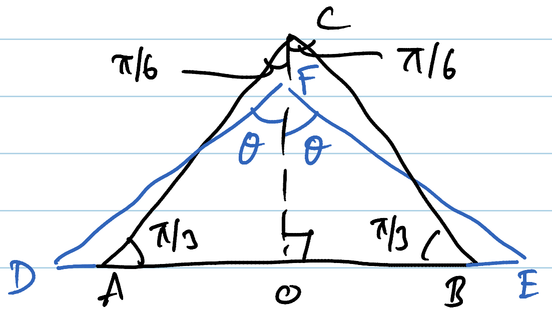
$\rightarrow \langle 110 \rangle$ direction.

Suppose a uniaxial load is applied along $\langle 110 \rangle$ direction. Then the separation between atoms along this direction increases. But this increase in separation implies higher energy state for the crystal.

$\therefore$ the atoms move in the lateral direction to reduce the separation. Thus, two things happen:

1) Increase in separation along $\langle 110 \rangle$ from r_0 to, say, r
2) Decrease in separation along the $\perp$ direction

The geometry can be better understood by looking at the three atoms I, II + III above. The configuration of these 3 atoms is as shown:



$\rightarrow \langle 110 \rangle$

DEF – position after deformation.

ABC – position before deformation.

$$\overline{AO} = \overline{BO} = r_0 \qquad \overline{AC} = \overline{BC} = 2r_0$$

$\overline{DE} = 2r_0$, as the hard spheres are in contact

Let the angle $\angle DFO = \theta$; $\angle ACO = \pi/6$

Since we are dealing with elastic deformations, θ is only slightly greater than $\angle ACO$.

$\therefore$ we can say $\theta = \angle ACO + \delta$, where $\delta << 1$

$$\Rightarrow \boxed{\theta = \pi/6 + \delta}$$

From the figure, we also know that

$$\sin\theta = \frac{r}{2r_0}$$

The axial strain $\varepsilon_a = \frac{r - r_0}{r_0} = \frac{2r_0 \sin\theta - r_0}{r_0}$

$$\Rightarrow \varepsilon_a = 2\sin\theta - 1$$

$$= 2\sin\left(\frac{\pi}{6} + \delta\right) - 1$$

$$= 2\left[\sin\frac{\pi}{6}\cos\delta + \cos\frac{\pi}{6}\sin\delta\right] - 1$$

$$\sim 2\left[\frac{1}{2} + \frac{\sqrt{3}}{2}\delta\right] - 1 \quad \left[\text{Since } \delta << 1, \cos\delta \simeq 1, \sin\delta \simeq \delta\right]$$

$$= \sqrt{3}\,\delta$$

Lateral strain $\varepsilon_l = \frac{\overline{OE} - \overline{OC}}{\overline{OC}} = \frac{r_0\cos\theta - \frac{\sqrt{3}}{2}r_0}{\sqrt{3}/2\, r_0}$

Now, $\cos\theta = \cos(\pi/6 + \delta)$

$$= \cos\frac{\pi}{6}\cos\delta - \sin\pi/6 \sin\delta$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{2}\delta$$

$= \frac{\sqrt{3}}{2} - \frac{1}{2}\delta$

$\Rightarrow \varepsilon_l = \frac{\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\delta\right) - \sqrt{3}/2}{\sqrt{3}/2} = -\delta/\sqrt{3}$

By definition, Poisson's ratio $\nu = \frac{-\varepsilon_l}{\varepsilon_a}$

$= -\left[\frac{-\delta/\sqrt{3}}{\sqrt{3}\delta}\right]$

$\Rightarrow \boxed{\nu = \frac{1}{3}}$

2) We are given that $\phi = 65^\circ$

a) $\lambda_1 = 30^\circ$, $\lambda_2 = 48^\circ$, $\lambda_3 = 78^\circ C$

Since $\cos\lambda_1$ is the highest, this direction is favoured for slip

b) $m_1 = \cos\lambda_1 \cos\phi = \cos 30^\circ \cos 65^\circ = 0.366$

Yield occurs when $m\sigma = \tau_{crss}$

$\Rightarrow \tau_{crss} = (0.366)(2.5\text{ MPa})$

$= 0.915\text{ MPa}$

3) $\tau_{crss} = 1\text{ MPa}$

$\lambda = 45^\circ$

$\sigma = 3.5\text{ MPa}$

$\Rightarrow (\cos 45^\circ \cos\phi)\, 3.5 = 1$

$\Rightarrow \cos\phi = \frac{\sqrt{2}}{3.5} \Rightarrow \phi = 66.2^\circ$

Let us say that slip occur on the (111) plane along the $\langle \bar{1}10 \rangle$ direction.
The above calculation shows that the $\langle 111 \rangle$ direction should make 66.2° with the loading direction.

This choice of slip system is not unique. Any of the 11 other equivalent choices might be made.

4. Length of Burgers vector for FCC crystal $= \frac{a}{\sqrt{2}} = \frac{0.36151 \text{ nm}}{1.414}$

$= 0.25566 \text{ nm}$ //

6. $$\tau = \tau_0 + \frac{k}{\sqrt{d}}$$

When $d = 10^{-5}$ m, $\tau = 230$ MPa

$d = 6 \times 10^{-6}$ m, $\tau = 275$ MPa

$$\Rightarrow \quad 45 \text{ MPa} = k \left[\frac{1}{\sqrt{6 \times 10^{-6}}} - \frac{1}{\sqrt{10^{-5}}} \right]$$

$$\Rightarrow \quad k = 0.489 \text{ MPa}\sqrt{\text{m}}$$

Then $$\tau_0 = 230 - \frac{0.489}{\sqrt{10^{-5}}} = 75.4 \text{ MPa}$$

Need to find d when $\tau = 310$ MPa

or $$310 = 75.4 + \frac{0.489}{\sqrt{d}}$$

$$\Rightarrow \quad \frac{0.489}{\sqrt{d}} = 236.6 \text{ MPa}$$

$$\Rightarrow \quad d = 4.34 \ \mu\text{m}$$