Tutorial #2, due in class on August 19, 2011

Ground Rules

- Consultation with your team-mates, TAs or the Instructor is encouraged. However, each student is expected to write out and hand in his/her own solutions.
- Please turn in solutions to all questions today.
- **Grading**: 1 point for fully correct solution. No partial grading.

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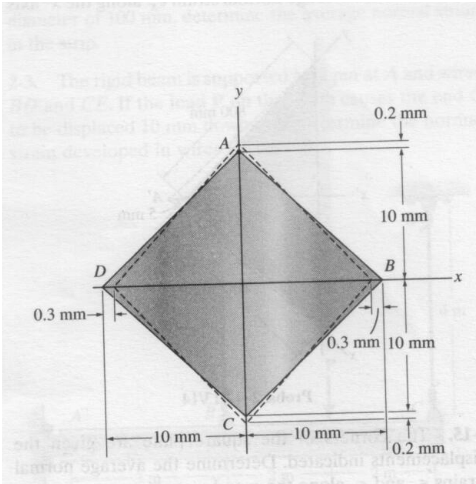
1. The corners of the square plate shown below are given the displacements indicated. Determine the average normal strains along side AB and diagonals AC and DB.



- **Solution:** The normal strains can be found by dividing the change in length by the original length of each side.
  - For AB,

$$\Delta L = \sqrt{10.2^2 + 9.7^2} - 10\sqrt{2} = -0.066\text{mm} \Longrightarrow \epsilon = -0.066/10\sqrt{2} = -0.0047$$

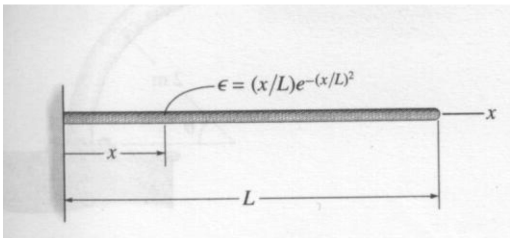
  - For AC,

$$\Delta L = 20.4 - 20 = 0.4\text{mm} \Longrightarrow \epsilon = 0.4/20 = 0.02$$

  - For BD,

$$\Delta L = 19.4 - 20 = -0.6\text{mm} \Longrightarrow \epsilon = -0.6/20 = -0.03$$

2. The wire shown below is subjected to a normal strain that is defined by $\epsilon = (x/L)\, e^{-(x/L)^2}$, where x is length measured in millimetres and L is the initial length. Determine the increase in the length of the wire.



- **Solution:** The total increase in length is given by

$$\Delta L = \int_0^L \epsilon(x)dx = \int_0^L \left(\frac{x}{L}\right) e^{-\left(\frac{x}{L}\right)^2} dx$$

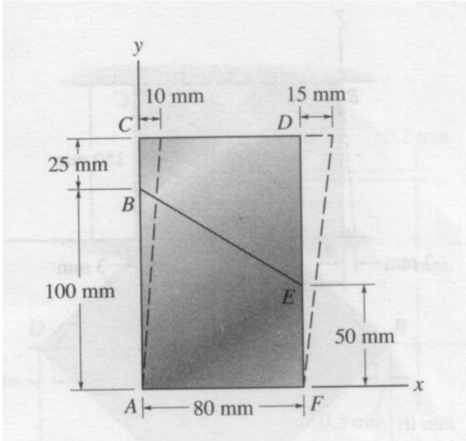
- This integral can be solved by making the substitution $t = \left(\frac{x}{L}\right)^2$. Then, $dt = 2\frac{x}{L^2}dx$ or $\frac{x}{L}dx = \frac{L}{2}dt$. Thus, the expression for ΔL becomes

$$\Delta L = \frac{L}{2}\int_0^1 e^{-t}dt = \frac{L}{2}\left(e^0 - e^{-1}\right) = \frac{L\,(e-1)}{2e}$$

3. A block of material distorts into the dashed position as shown in the figure below. Determine

(a) the average strains $\epsilon_{xx}, \epsilon_{yy}, \epsilon_{\text{xy}}$ at A and

(b) the average normal strain along line BE.



- **Solution:**

(a) At A, $\epsilon_{xx} = 0$ because there is no change in length along the x-direction.
The element originally oriented along the y-direction was originally of length 125 mm, and is of length $\sqrt{125^2 + 10^2} =$ 125.4mm. Thus, $\epsilon_{yy} = \frac{0.4}{125} = 0.0032$.
The average shear strain ϵ_{xy}at A is merely one-half the **reduction** in angle between line elements orignally aligned along the x- and y-axes. From the figure, $\epsilon_{xy} = 0.5 \times \tan^{-1}\left(\frac{10}{125}\right) = 0.04$

(b) The original length of BC is $\sqrt{50^2 + 80^2} = 94.34$mm. After deformation, point B is displaced $10 \times \frac{100}{125} = 8$mmto the right and point E is displaced $15 \times \frac{50}{125} = 6$mmto the right. Thus, the new length of BC is $\sqrt{50^2 + 78^2} =$ 92.65mm. Hence, the strain along BC is $\epsilon = \frac{-1.69}{94.34} = -0.0179$.