Tutorial #4, due in class on Sep 5, 2011

Ground Rules

- Consulta
- tion with your team-mates, TAs or the Instructor is encouraged. However, each student is expected to write out and hand in his/her own solutions.
- Please turn in solutions to all questions today.
- **Grading**: 1 point for fully correct solution. No partial grading.

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1. Electrical resistance strain-gages are used to measure surface strains. Any strain causes a change in the length of the foil used in the strain gage. Knowing the change in resistance and the resistivity of the foil, one can then calculate the strain. Three strain gages are usually attached in a 'rosette' to measure the axial surface strain along three directions. In this arrangement, the second gage is inclined at 45^o c.c.w. to the first and the third is at 90^o c.c.w. to the first. Let the three measured strain values be ϵ_a, ϵ_b and ϵ_c.

   (a) If the principal surface strains are ϵ_1 and ϵ_2 and the first principal direction is inclined at θ to the first strain gage, derive expressions for ϵ_1, ϵ_2 and θ in terms of the measured strains.
   Solution: Since there are no surface tractions applied, we have a state of plane stress. We are given that the first principal strain direction is inclined at angle *theta* to the first strain gage.

   Let us work with the principal directions. In this coordinate system, the matrix of strain components is :

$$\begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix}$$

   Using the standard coordinate transformation, the axial strain at angle ϕ (c.c.w.) from $\mathbf{e}_1$ is given by

$$\epsilon' = \epsilon_1 \cos^2 \phi + \epsilon_2 \sin^2 \phi = \frac{\epsilon_1 + \epsilon_2}{2} + \frac{\epsilon_1 - \epsilon_2}{2} \cos 2\phi \tag{1}$$

   The first, second, and third strain gages are at angles $-\theta$, $\pi/4 - \theta$, and $\pi/2 - \theta$ from $\mathbf{e}_1$. Substituting these angles From equation (1) with $\phi = -\theta$, we get

$$\begin{aligned} \epsilon_a &= \tfrac{\epsilon_1+\epsilon_2}{2} + \tfrac{\epsilon_1-\epsilon_2}{2} \cos 2\theta \\ \epsilon_b &= \tfrac{\epsilon_1+\epsilon_2}{2} + \tfrac{\epsilon_2-\epsilon_1}{2} \cos 2\theta \\ \epsilon_c &= \tfrac{\epsilon_1+\epsilon_2}{2} + \tfrac{\epsilon_1-\epsilon_2}{2} \sin 2\theta \end{aligned}$$

   Solving these three equations for θ, we get

$$\begin{aligned} \tan 2\theta &= \tfrac{2\epsilon_b-(\epsilon_a+\epsilon_c)}{\epsilon_a-\epsilon_c} \\ \epsilon_1 &= \tfrac{\epsilon_a+\epsilon_c}{2} + \sqrt{(2\epsilon_b - (\epsilon_a + \epsilon_b))^2 + (\epsilon_a - \epsilon_c)^2} \\ \epsilon_1 &= \tfrac{\epsilon_a-\epsilon_c}{2} + \sqrt{(2\epsilon_b - (\epsilon_a + \epsilon_b))^2 + (\epsilon_a - \epsilon_c)^2} \end{aligned}$$

   Note that the correct value of theta (of the two solutions available) is chosen based on the signs of the numerator and the denominator.

   (b) What is the third principal strain?
   Solution: The third principal strain can be evaluated from the condition $\sigma_{33} = 0$ (plane stress).

$$\epsilon_3 = \frac{-\nu}{1-\nu}(\epsilon_a + \epsilon_c)$$

2. What is the volume change of a 10-cm diameter copper ($E = 120$ GPa) sphere subjected to a fluid pressure of 20 MPa?
   Solution:
   Volumetric strain $= \epsilon_{kk} = \sigma_{kk} \frac{1-2\nu}{E}$ Assume $\nu = 0.3$, a typical value for metals.

$$\sigma_k k = -3p = -60MPa$$

   Then,

$$\epsilon_{kk} = -60 \times 10^6 (1 - 2 * 0.3)/60 \times 10^9 = 2 \times 10^{-4}$$

   .

   The volume change $= \epsilon_{kk} * V = 2 \times 10^{-4} 4\pi 0.05^3/4 = 0.0785$cc.

3. Determine the engineering strain e, the true strain ϵ and the reduction in area $q = 1 - \frac{A_f}{A_0}$ for each of the following situations:

   (a) Extension from L to $1.1L$
   (b) Compression from h to $0.9h$
   (c) Extension from L to $2L$
   (d) Compression from h to $0.5h$
   (e) Compresion to zero thickness

   - **Solution:** $e = \frac{L_f - L_i}{L_i}$, $\epsilon = \log \frac{L_f}{L_i}$, $q = 1 - \frac{A_f}{A_0}$

	e	ϵ	q
a	0.10	0.095	0.0909
b	-0.10	-0.105	-0.111
c	1.0	0.693	0.50
d	-0.5	-0.693	-1.0
e	-1.0	$-\infty$	undefined