

Tutorial #8, due in class on Nov. 4, 2011

1. A thin plate of steel contains a central through-thickness flaw of length 16 mm, which is subjected to a stress of 350 MPa applied perpendicularly to the flaw plane. The 0.2% flow stress of the material is 1400 MPa.

Calculate the plastic zone size and the effective stress intensity level at the crack tip, making reasonable assumptions about the state of stress.

- **Solution:** Since the plate is thin, we will assume a state of plane stress and we will verify this assumption afterwards. The plastic zone size under plane stress is given

$$r_y = \frac{1}{2\pi} \left(\frac{K_I}{\sigma_y} \right)^2$$

- The case at hand is that of a panel with a central crack of length $2a$; $K_I = \sigma\sqrt{\pi a} = 350\sqrt{3.14 \times 0.008\text{m}} = 55.5\text{MPa}\sqrt{\text{m}}$. With this value, we get $r_y = 0.25\text{mm}$. Since we can reasonably expect a sheet to be a few mm thick, we see that the plane stress assumption is not correct in this case. Instead, if we assume plane strain conditions to prevail, we get $r_y = 0.08\text{mm}$, which will be less than one-tenth of the sheet thickness, thus justifying our assumption of plane strain.
- Note also that here the crack length is 200 times the plastic zone size, which means that LEFM can be safely applied.
- Due to the presence of the plastically deformed region ahead of the crack tip, the sheet is more compliant, just as it would be if the crack were longer. Irwin argued that the effective crack length is $2(a + r_y)$ (the tip is centered in the plastic zone of size $r_p = 2r_y$). Thus, the effective crack length here is 16.16 mm.

If, after heat treatment, the flow stress of the steel dropped to 385 MPa, what would the plastic zone size be under the applied stress of 350 MPa, and what conclusions would you draw about the use of LEFM?

- **Solution:** Now, for plane stress, $r_y = 3.3\text{mm}$ and for plane strain, $r_y = 1.1\text{mm}$. The former is now a valid assumption since the plastic zone size is now likely greater than the thickness of the sheet. But, now the plastic zone size is comparable to the crack size, and the use of LEFM is not justified.
2. Suppose that an ASTM compact tension specimen is used to measure the fracture toughness of a steel. The specimen has dimensions $W = 40\text{mm}$ and $B = 20\text{mm}$. The crack length was 18.5 mm, and the fracture load was 15kN. For a compact tension specimen, the stress intensity factor is given by

$$K = \frac{P}{B} \sqrt{\frac{\pi}{W}} \left[16.7 \frac{a^{(1/2)}}{W} - 104.7 \frac{a^{(3/2)}}{W} + 369.9 \frac{a^{(5/2)}}{W} - 573.8 \frac{a^{(7/2)}}{W} + 360.5 \frac{a^{(9/2)}}{W} \right]$$

- (a) Calculate K_c of the steel.

- **Solution:** From the numbers given, $K_c = 32.2\text{MPa}\sqrt{\text{m}}$.

- (b) If the steel has yield stress 800MPa, was this a valid fracture toughness measurement?

- **Solution:** The fracture test is valid if both

$$B \text{ and } a \geq 2.5 \left(\frac{K_{IC}}{\sigma_Y} \right)^2 = 2.5 \times \left(\frac{32.2}{800} \right)^2 = 0.004\text{m} = 4\text{mm}$$

- Since this is satisfied, this is a valid fracture toughness test.

3. A sharp penny-shaped crack with a diameter of 2.5 cm is completely embedded in a solid. Catastrophic fracture occurs when a stress of 700 MPa is applied.

- (a) What is the fracture toughness of the material? (Assume that this value is for plane-strain conditions)

- **Solution:** $K_{IC} = \frac{2}{\pi}\sigma\sqrt{\pi a} = 88.3\text{MPa}\sqrt{\text{m}}$

- (b) If a sheet (0.75 cm thick) of this material is prepared for fracture-toughness testing ($t = 0.75$ cm, $a = 3.75$ cm), would the K_c value be a valid fracture toughness number (the yield strength of the material is given to be 1100 MPa)

- **Solution:**

$$2.5 \left(\frac{K_{IC}}{\sigma_Y} \right)^2 = 2.5 \left(\frac{88.3}{1100} \right)^2 = 1.6\text{cm}$$

- Since this value is greater than the thickness of the sheet, this will not be a valid fracture toughness test.

- (c) What would be a sufficient thickness for valid K_{IC} determination?

- 1.6cm