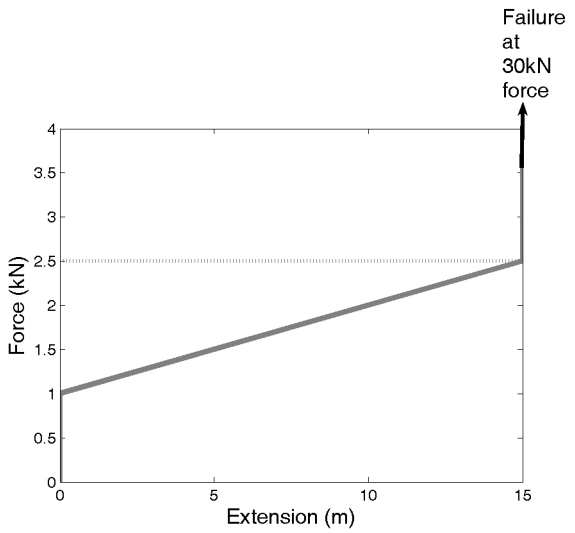
Quiz #1, Sep 25 2009

Instructions

1. If something is not clear, come up and ask in a soft voice; do **not** discuss amongst yourselves.
2. Time limit is 50 minutes, although this will not be strictly enforced.
3. Cheating of any sort will not be tolerated; you will receive zero credit and may face additional consequences.

Questions

1. Compute the value of the Poisson's ratio of an incompressible, isotropic, linear elastic material. (Do not just quote the answer if you know it, please derive it). (1)
2. Show that for a linear, elastic material, the principal directions of the stress and strain tensor coincide (1). What is the relationship between their eigenvalues? (1)
3. A material used to manufacture bungee cords has the following load-displacement curve. The initial length of the rope is 15m. One end of the rope is attached to a bridge deck and the other end is attached to the jumper standing on the deck. The jumper then jumps off the deck and descends vertically until arrested by the rope.



(a) How far does a jumper of mass 50kg fall before coming to a stop? (2)

(b) What is the maximum mass of a jumper that can be successfully arrested? (1)

4. For thin cylindrical vessels with end caps subjected to internal pressure, stresses are conveniently described using cylindrical coordinates (r, θ, z); r the radial distance from the cylinder axis, θ the angle measured positive counter-clockwise along the circumference of the cylinder, and z the axial coordinate along the cylinder axis. If the internal pressure is equal to p and external pressure is negligible,

   (a) What are the radial stresses at the inner and outer walls of the cylinder? (1)

   (b) Away from the ends, the circumferential or hoop stress, and the axial stress are respectively approximated as

   $$\sigma_\theta = \frac{pa}{h}, \quad \sigma_z = \frac{pa}{2h}$$

   where a is the inner radius of the cylinder and h is the thickness of the cylinder wall. Calculate the deviatoric stress components. (1)

   (c) If the cylinder material obeys von Mises plasticity with a yield stress in tension equal to σ_0, calculate the critical value of internal pressure p_{cr} that will cause yielding for a given pressure vessel geometry. (2)

A useful formula

$$\sigma_{ij} = \frac{E}{1+\nu}\left[\epsilon_{ij} + \frac{\nu}{1-2\nu}\epsilon_{kk}\delta_{ij}\right]$$